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Review on Graph Theory Applications in Data Analysis and Network Design

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Abstract: *Numerous mathematical theorems and models are validated through the application of graph theory. In this paper, the diverse applications and methods of graph theory are presented to address issues in a variety of scientific and technological disciplines, in addition to mathematics. Almost any physical situation that involves discrete objects and a relationship among them can be represented using a graph. This abstract offers a succinct summary of the fundamental principles of graph theory, which encompass crucial theorems, fundamental terminologies, and graph types. Additionally, it emphasizes the practical applications of graph theory, including optimized network flows, graph coloring problems, and shortest path algorithms. This abstract endeavors to cultivate a more profound comprehension of the influence of graph theory on contemporary computational paradigms and problem-solving methodologies by elucidating its complexities. These disciplines encompass operations research, software engineering, computer science, biology, chemistry, and website design.*

Keywords: Graph Theory, Network Analysis, Combinatorics, Algorithm Design

I. INTRODUCTION

The study of graphs, which are mathematical structures made up of vertices linked by edges, is the focus of graph theory, a fundamental area of discrete mathematics. With vertices standing in for things and edges for the connections or interactions between them, graphs are used in graph theory to explain relationships between items. Examining a variety of graph attributes, including connectedness, pathways, cycles, and coloring, is part of the study of graph theory. Additionally, it looked at directed graphs and other graph types. In a variety of fields, such as computer science, operations research, biology, communications, and the social sciences, graph theory offers a strong framework for issue analysis and resolution.

It provides a variety of methods and algorithms for resolving issues with routing, optimization, modeling, and network connection. All things considered, graph theory is an essential tool in discrete mathematics that provides understanding of the composition and behavior of intricate systems represented by graphs and makes it possible to solve practical issues in a variety of fields. Graphs are used to show communication networks, organize data, and determine the shortest route in a network or a road. Roads are shown as edges on Google Maps, while other locations are represented as vertices or nodes. The shortest route between two nodes is determined using graph theory. The study of graphs, the primary focus of discrete mathematics, is known as graph theory in mathematics. Any graph's vertices are connected by its edges. In addition to discrete mathematics, linear graphs may be used in computer science, biology, linguistics, physics, chemistry, and other fields. Since GPS is used to monitor routes and determine the direction of roads, it is the finest real-world illustration of graph structure.

History

In discrete mathematics, graph theory has its roots in the 18th century, when mathematicians such as Leonard Euler investigated the well-known Seven Bridges of Konisberg issue in 1736. By establishing the idea of a graph as a mathematical abstraction to describe and analyze interactions between things, Euler's answer to this issue established the groundwork for graph theory. However, significant formal advancements in graph theory did not appear until the 19th century. Gustav Kirchhoff laid the foundation for the study of network theory in 1847 when he proposed the idea

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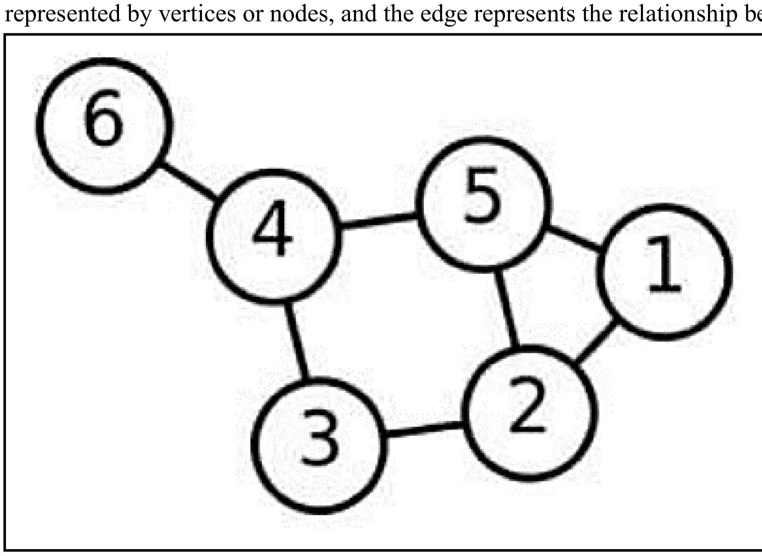
of a graph in relation to electrical circuits. Arthur Cayley developed the study of trees and the idea of graph isomorphism later in the century.

Mathematicians like Percy MacMahon, who researched graph enumeration issues, and William Hamilton cycles made important contributions to graph theory in the late 19th and early 20th century. Graph theory saw a boom in attention and advancement in the middle of the 20th century, especially with the introduction of computers. Notable contributions to graph theory were made by mathematicians and computer scientists such as Claude Shannon, Paul Erdos, and Edges Dijkstra, who created methods and algorithms for resolving graph-related issues. Graph theory continued to advance quickly throughout the latter part of the 20th century and the early years of the 21st, finding use in a wide range of disciplines, including computer science, operations research, biology, telecommunications, and the social sciences. Graph theory is becoming a vital tool in contemporary science and mathematics due to the creation of novel algorithms, theories, and applications.

According to its history, graph theory was first presented by the renowned Swiss mathematician Leonard Euler, who used it to build graphs from provided data or a collection of points in order to solve several mathematical problems. Bar graphs, frequency tables, line graphs, circle graphs, line plots, and other forms of data are all shown graphically. The Konisberg bridge issue in 1735 marked the beginning of graph theory. Euler researched the Konisberg bridge issue and developed a structure known as an Eulerian graph to address it. A.F. Mobious provides the bipartite and full graph information. Kuratowski used leisure tasks to demonstrate their planerness in 1840. In 1845, Gustav Kirchhoff introduced the idea of a tree, which is used to calculate currents in electrical networks or circuits. In 1852, Thomas Gutherie discovered the four-color issue. After discussing the cycle and polyhydra, William Hamilton and P. Kirkman came up with the idea of the Hamiltonian graph. H. Dudeney brought up a puzzling issue in 1913. Sylvester coined the word "graph" in 1878.Another area of graph theory known as extreme graph theory was discovered in 1941 as a result of Ramsey's work on the idea of collarations. Heinnch used a computer to solve the four-color issue in 1969.

Preliminaries

Graph: A graph with two sets of vertices (V) and edges (E) is represented as G(V,E). A graph is an organized visual representation of any data in mathematics. The link between variable quantities is shown in the graph. According to graph theory, a graph is a collection of items that are connected to one another in some way. In essence, the objects are mathematical notions represented by vertices or nodes, and the edge represents the relationship between the two nodes.



Basic Terminology

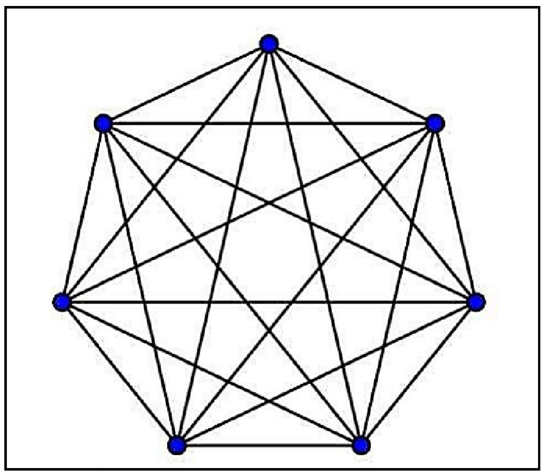
[Trivial Graph] A graph consisting only one vertex and no edge.

[Null Graph] A graph consisting n vertices and no edge.

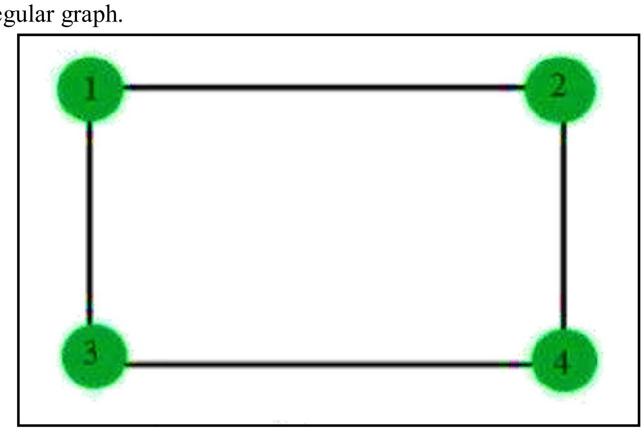
[Directed Graph]: A graph consist the direction of edges then this is called di directed graph.
[Simple Graph] A graph does not contain any self loop and multiedge.
[Multigraph]A graph does not contain any self loop but contain multiedge is called multigraph.
[Isolated Vertex Pendant Vertex] A vertex having 0 is called isolated Vertex and a vertex having degree 1 is called pendant vertex.
[VFinite and Infinite graph] A graph with a finite number of vertices as well as edges is called finite graph otherwise it is an infinite graph.
[Pseudo Graph] A graph contain both self loop and multiedge is called pseudo Graph.
[Undirected Graph] A graph which is not directed then it is called undirected graph.
[Self loop in a Graph] if edge having the same vertex as both its end vertices is called self loop.
[Proper edge] An edge which is not self loop is called proper edge.
[Multi edge] A collection of two or more edges having identically end point.

Some Important Graphs

[Complete Graph] A simple connected graph is said to be complete if each ver-tex is connected to every other vertex



[Regular Graph] A graph G is said to be regular if every vertex has the same degree. If degree of each vertex of graph G is K, then it is called k-regular graph.



[Bigraph (or Bipartite)] If the vertex set V of a graph G can be partitioned into two non- empty disjoint subsets X and Y in such a way that edge of G has one end in X and one end in Y. Then G is called bipartite.

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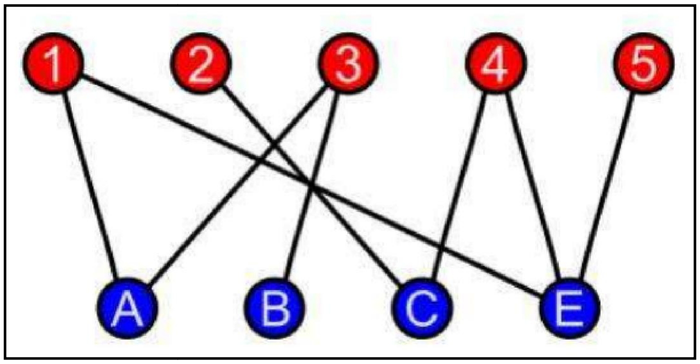
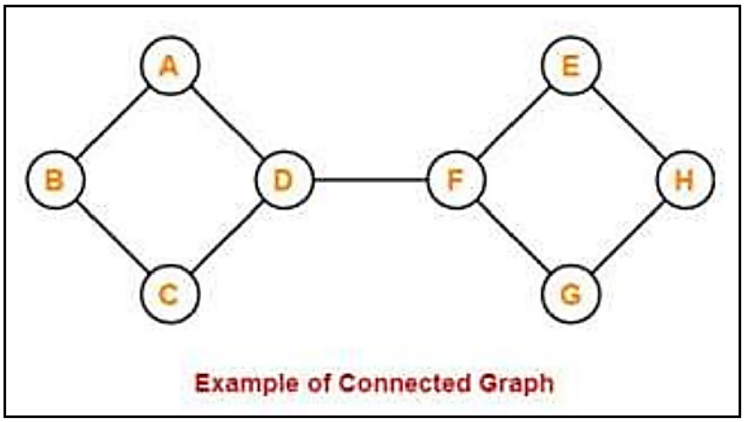


[CONNECTED GRAPH] An undirected graph is said to be connected if there is a path between every two vertices.

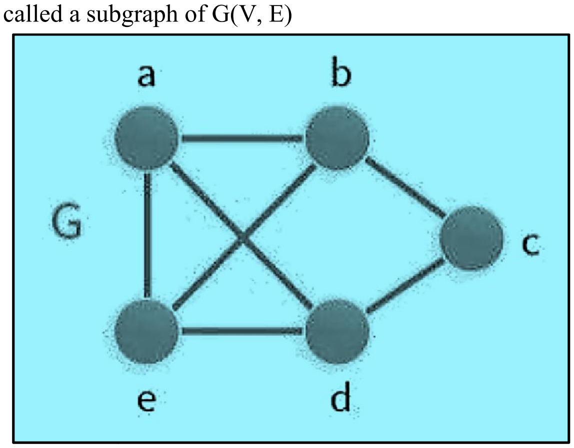


Example of Connected Graph

Remarks: If a graph is connected then it will not bipartite

[Complete Bipartite Graph] : If every vertex in X is disjoint is every vertex in Y, then it is called a complete bipartite graph. If X and Y condition m n vertices then this graph is denoted by vn.

[Subgraph] Let G(V, E) be a graph. Let V' be a subset of V and let E' be a subset of E whose end point belong to V'. Then G(V' , E') is a graph and called a subgraph of G(V, E)



Decomposition of graph

A graph is said to be decomposed into two subgraph G1 G2 if G1G2 = G G1G2 null graph.

Complement of graph

The complement of a graph G is defined as a simple Graph with the same vertex set as G and where two Vertices u and v are adjacent only when they are not adjacent in G.

Planare graph

A graph which can be drawn in the plane so that it's Edges do not cross is called planare.

Rooted tree

A rooted tree is a tree in which one vertex is root.

Binary tree

A binary tree is defined as a tree in which there is exactly one vertex of degree two and each of remaining vertices is of degree one or three and vertex of degree two is serves as a root.

Pendent vertex in tree

A vertex of degree one is called pendent vertex of tree.

Path length of tree

path length of a tree is defined as the sum of edges from the root of all pendent vertices.

Spanning tree

If G is any connected graph a spanning tree in G is a subgroup T of G, which is a tree.

Eulerian path

A path in a graph is said to be an eulerian path if it traverses each edge in the graph once and only once.

Eulerian circuit

A circuit in a graph is said to be an eulerian circuit if it traverses each edge in the graph once and only once.

Eulerian grap

A connected graph which contains an eulerian circuit is called eulerian graph.

Hamiltonian path

A path which contains every vertex of a graph G exactly once is called Hamilto-nian graph.

Hamiltonian circuit

A circuit that passes through each of the vertices in a group G exactly one except the starting vertex and end vertex is called Hamiltonian circuit.

Hamiltonian graph

A connected graph which contain Hamiltonian circuit is called Hamiltonian graph.

Weighed graph

A graph is called weighed graph if a non-negative integer w(e) associate to each edge and this w(e) is a weight of corresponding edge.

Chromatic Number

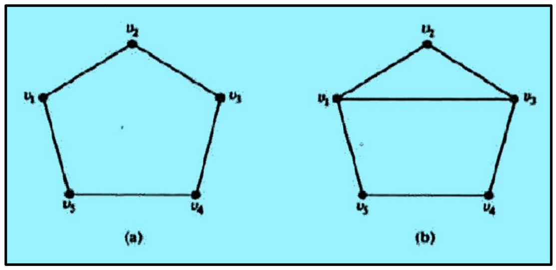
The least number of colors required for coloring of a graph G is called it's chro-matic number.

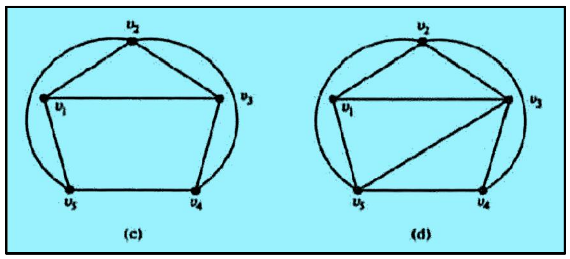
Kuratowski's Theorem

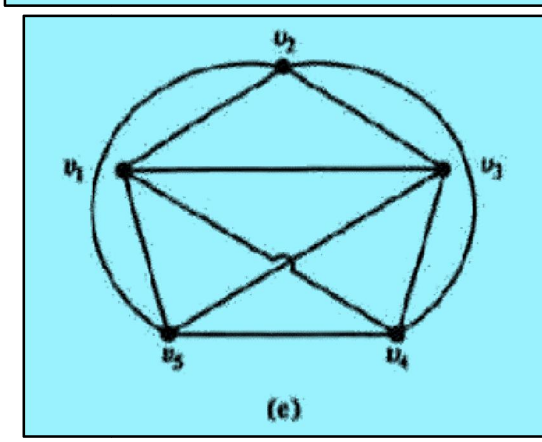
A key finding in graph theory that describes planar graphs is Kuratowski's Theorem. A graph is said to be planar if and only if it does not have a subgraph that is homeomorphic to either the full bipartite graph k3,3 (the complete bipartite graph with two sets of three vertices each) or the complete graph k5, which is the complete graph with five vertices. In other words, if and only if a graph does not include a subgraph that resembles K5 or K3,3 then it may be shown on a plane without any edges crossing one another. Knowing which graphs are planar and which are not is obviously important since planarity is such a basic characteristic. Specifically, Ks and K3•3 are nonplanar, and every valid subgraph of any of these graphs is planar, as we have previously shown. Kuratowski (1930) characterized planar graphs in a fairly straightforward manner.

Theorem: The complete graph of five vertices is nonplanar.

Proof: Let v1, v2, v3, v4, and v5 be the names of the five vertices in the whole network. A simple graph that has edges connecting each vertex to every other vertex is called a full graph. We need a pentagon-shaped circuit that runs from v1 to v2 to v3 to v4 to v5 back to v1. The Jordan curve theorem requires that this pentagon split the paper's plane into two sections, one inside and one outside. Since an edge from vertex v1 to v3 is required, this edge may be drawn within or outside the pentagon. Let's say we decide to draw a line within the pentagon from v1 to v3. Drawing an edge from v2 to v4 and another from v2 to v5 is now necessary. We draw both of these edges outside the pentagon since it is impossible to draw any of them within the pentagon without going over the already-drawn edge. It is impossible to draw the boundary between v3 and v5 outside the pentagon without going over the border between v2 and v4. As a result, an edge within the pentagon must link v3 and v5. We still haven't drawn a line between v1 and v4. Without a crossover, this edge cannot be positioned inside or outside the pentagon. As a result, a plane cannot contain the graph. The first of Kuratowski's two graphs is a complete graph with five vertices. With six vertices and nine edges, Kuratowski's second graph is a regular connected graph. It is rather simple to see that the graphs are isomorphic.







Theorem: Kuratowski's second graph is also nonplanar

Proof: Several properties common to the two graphs of Kuratowski. These are

Both are regular graphs.

Both are nonplanar.

Removal of one edge or a vertex makes each a planar graph.

Kuratowski's first graph is the nonplanar graph with the smallest number of vertices, and Kuratowski's second graph is the nonplanar graph with the Smallest number of edges. Thus both are the simplest nonplanar graphs. In the literature, Kuratowski's first graph is usually denoted by K5 and the Second graph by K3,3 letter K being for Kuratowski.

Application of graph theory

Graph theory, a branch of discrete mathematics, has numerous application in vari-ous fields.

Computer Network

Graphs are used to represent networks of communication, Data organisation, computational devices, the flow of computation etc. One practical example is the link structure of a website could be represented by a directed graph

Networks

Graph theory is extensively used in the study of networks, such as social Net-works, transportation networks, communication networks and computer

Networks

It helps in analysing connectivity, identifying critical nodes, optimising routes, And understanding networks resilience.

Computer science

Graph theory plays a vital role in com-puter science, particularly in the design and Analysis of algorithms. It is used in data structures Like adjacent lists for representing graphs. Algorithms like Dijk-stra's algorithm for shortest Paths, Prim's algorithm for minimum Spanning trees, and algorithms for graph Traversal are all based on graph theory.

Circuit Design

In electrical engineering, graph theory is ap-plied to analyze and Design circuits.Graph models help in representing connections between Components and analysing properties like voltage distribution, current flow and circuit efficiency.

Operations Research

Graphs are used to model optimization problems like the traveling salesman problem, where the objective is to find the shortest routes that visits a set of cities exactly once and returns to the origin City.

Biology

Graph theory is applied in bioinformatics for modelling molecular structures, proteinprotein interactions, and genetics networks.

Chemistry

Graph theory is used to model and analyze molecular structures, chemical reac-tions, and chemical bonding patterns.

Transportation

Graph theory is used to model transportation networks and analyze traffic flow, optimising routes, and minimising congestion.

Operation Management

Graph theory is applied in project management to model project activities, de-pendencies, and critical paths.

Epidemiology

Graph theory plays a crucial role in modelling the spread of diseases. Nodes represent individuals, and edges represent contacts or interactions between them. Epidemiologists use graph algorithms to simulate dis-ease transmission, identify key influencers, and develop strategies for diseases control.

Power Grids

Graph theory is employed in modelling and analyzing electrical power grids. Nodes represent power stations, substations, and consumers, while edges represent transmission lines. Graph algorithms help in optimizing power flow identifying vul-nerabilities, and designing resilient power grid systems.

Game Theory

Graph theory is applied in graph theory to model strategic interactions between players in various games, including social dilemmas, voting systems, and economic competitions. Graph algorithms help analyze equilibrium strategies, coalition form-ations, and game dynamics.

Genetics

Graph theory is used in genetics for analyzing genetic networks, genome se-quencing, and evolutionary relationship between species. Graph algorithms help in ntifying genetic patterns, predicting gene functions, and understanding genetic diseases.

Geography

Graph theory is used in geographic information systems (GIS) for spatial ana-lysis, route planning, and mapping. It helps in analyzing spatial relationships and optimizing geographic data .

II. CONCLUSION

In discrete mathematics, graph theory is essential because it provides strong ideas and techniques for examining and resolving a variety of issues. Its study helps researchers and mathematicians understand the features, connection, and structure of discrete and network systems. Graph theory is a diverse and essential topic of study because of its applications in a wide range of disciplines, including computer science, biology, telecommunications, geography, and the social sciences. Our knowledge of complex systems and networks in the digital era and beyond is shaped by graph theory's deep theoretical underpinnings and real-world applications, which continue to spur new discoveries and advancements.

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