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Mathematical Modelling of the Addiction of Drug Substances and Rehabilitation Control

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Abstract*: Drug addiction is a critical public health concern that affects the physical, psychological, and social well-being of individuals and communities. In this study, a deterministic nonlinear compartmental model is developed to analyze the dynamics of drug abuse and addiction, incorporating control interventions through public awareness and rehabilitation strategies. The mathematical model is rigorously analyzed to establish the positivity and boundedness of solutions, ensuring the biological feasibility of the model. Equilibrium analysis identifies two steady states: the Drug-Free Equilibrium (DFE) and the Drug Endemic Equilibrium Point (DEEP). Stability analysis reveals that the DFE is locally and globally asymptotically stable when the basic reproduction number R0<1R_0 < 1R0 <1, indicating the potential for drug eradication under effective control. Conversely, the DEEP becomes globally asymptotically stable when R0>1R_0 > 1R0 >1 and specific mortality parameters δ1=δ2=0\delta_1 = \delta_2 = 0δ1 =δ2 =0, as proven using a nonlinear Lyapunov function of the Go-Volterra type. Numerical simulations further validate the theoretical results, highlighting the critical roles of the awareness rate among susceptible individuals and the rehabilitation rate of addicted individuals in curbing the prevalence of drug addiction. The results suggest that coordinated implementation of awareness campaigns and effective rehabilitation programs can significantly reduce addiction levels and stabilize society toward a drug-free state.*

Keywords: Mathematical Modeling; Drug Addiction; Public Awareness; Rehabilitation Control; Basic Reproduction Number; Equilibrium Analysis; Lyapunov Stability

I. INTRODUCTION

Drugs are chemical substances that can alter the physiological and psychological functioning of a living organism. While many of these substances are medically prescribed for the treatment of various health conditions, their non-medical use—particularly without a prescription—can result in serious health, social, and legal consequences. This phenomenon, broadly referred to as drug abuse, encompasses the illicit, excessive, or inappropriate consumption of psychoactive substances, often leading to addiction, violence, and societal degradation (Matonya et al., 2021).

Recent trends indicate an alarming global increase in drug abuse, particularly among adolescents and young adults. Stimulants such as amphetamines—commonly prescribed for attention deficit disorders—are being misused alongside substances like cannabis, cocaine, synthetic opioids, alcohol, nicotine, and methaqualone. According to the World Health Organization (WHO, 2011), approximately 271 million people globally—representing 5.5% of the population aged 15–64—reported drug use in 2017. This represents a 30% increase from 2009. The prevalence of opioid and cannabis abuse has risen sharply across North and South America, Asia, and Africa, largely attributed to both population growth and evolving substance accessibility (United Nations, 2019).

In Africa, cannabis remains the most frequently abused illicit drug. West and Central Africa exhibit the highest regional prevalence, ranging from 5.2% to 13.5% (WHO, 2023). There is also growing misuse of amphetamine-type stimulants (ATS), such as “ecstasy” and methamphetamine. Additionally, the abuse of benzodiazepines, inhalants, and injectable narcotics among youth is increasingly reported, with a Sierra Leone survey indicating that 3.7% of youth use injectable substances (WHO, 2023). In the United States, data from the Centers for Disease Control and Prevention (CDC) in 2015 revealed a 63% increase in heroin snorting between 2002 and 2013, accompanied by a threefold surge in heroin-related deaths (Li et al., 2018). Similarly, in China, the number of registered drug users rose by 14.6% in 2015, reaching over 2.3 million individuals (Li et al., 2018).

In Nigeria, the escalating rate of youth incarceration has been closely linked to drug abuse and related criminal activities (Madaki et al., 2023). In response, the Nigerian government established the National Drug Law Enforcement Agency (NDLEA) and the National Agency for Food and Drug Administration and Control (NAFDAC) in 1990 to combat the growing substance abuse crisis (WHO, 2004).

To better understand and control the spread of drug addiction, several mathematical models have been proposed in recent years:

Salisu et al. (2021) developed a deterministic, nonlinear system of differential equations to model the co-dynamics of drug abuse and drug-induced violence. Their simulations demonstrated that targeted rehabilitation of violent and low-level drug users significantly reduced both phenomena, emphasizing the importance of integrated social interventions.

Anggriani et al. (2021) proposed an optimal control model, compartmentalizing the population into five classes: susceptible individuals (S), users of light-grade drugs (A), heavy-grade drug users (H), individuals receiving medication (T), and recovered individuals (R). Using Pontryagin's Minimum Principle and forward-backward sweep methods, they concluded that combining anti-drug campaigns with psychological counseling could effectively suppress drug addiction.

Mushanyu et al. (2016) modeled the influence of limited rehabilitation capacity using the Hill function in a nonlinear ODE system. Their findings underscored the significance of preventing relapse and enhancing community resources to manage rehabilitation effectively.

Kamiran et al. (2023) conducted a stability analysis of a drug rehabilitation model implemented in East Java. Their results indicated that rapid rehabilitation uptake could lead to faster attainment of a drug-free state in the population.

Mushanyu et al. (2018) introduced a sex-structured compartmental model to investigate gender-specific trends in drug addiction. Their analysis projected a gradual increase in female drug abusers in Cape Town, while male addiction rates were expected to decline, predicting that by 2030, 34% of women in specialized facilities would be undergoing treatment.

Eguda et al. (2022) explored the effects of addiction on youth productivity and societal contribution using a five-compartment model, including susceptible (Sh), drug users (Dh), addicted individuals (Ah), disabled individuals (Qh), and recovered individuals (Rh). Their analysis concluded that drug addiction among youth would remain stable if the effective reproduction number remained below one, highlighting the importance of early intervention.

Inspired by the existing body of literature, this study proposes a mathematical model that integrates public awareness programs and rehabilitation interventions to simulate and control drug addiction dynamics. By analyzing equilibrium points, computing the basic reproduction number R0R_0R0 , and performing stability analysis, the model aims to identify conditions under which drug addiction can be eradicated or managed effectively. Through numerical simulations, we further evaluate the impact of awareness and rehabilitation efforts on curbing addiction and guiding society toward a drug-free equilibrium.

Framework Organization

To understand the dynamics of drug addiction in a population, the entire population is stratified into six distinct compartments:

S: Susceptible individuals Secret drug users

: Public drug users

A: Drug-addicted individuals

Individuals undergoing rehabilitation, Recovered individuals Compartment Descriptions

Susceptible individuals (S): These are individuals who are vulnerable to drug use but have not yet used drugs.

Secret drug users: These individuals use drugs covertly due to social fear or stigma, yet they are already addicted.

Public drug users : These individuals use drugs openly, without social inhibition, and are also addicted.

Addicted individuals (A): This class includes those who progressed from either $_1$ or into chronic addiction. Rehabilitation individuals : This group comprises addicted individuals undergoing treatment or rehabilitation.

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Recovered individuals : These are individuals who successfully recovered either from rehabilitation or via awareness campaigns from secret and public use stages. Model Assumptions and Dynamics

Susceptible individuals are recruited at rate π.

They leave the susceptible class at rate λ, where

$$\lambda = \frac{\beta A S}{N}$$

Here, β is the effective contact rate, A is the number of addicts, and N is the total population.

A fraction p of susceptible individuals transitions into secret drug users ($_1$) at rate pλ, and the remaining fraction (1 - p) enters the public user class ($_2$) at rate (1 - p)λ.

Transition Rates:

Decreases due to awareness at rate a and progression to addiction at rate σ_1. Progresses to addiction at rate σ_2.

A: Increases by inputs from and ; decreases due to drug-induced mortality (δ_1) and rehabilitation (γ).

Increases from A at rate γ, and decreases due to recovery (τ) and mortality (δ_2). Increases from recovery of and by awareness of; decreases by natural mortality (μ).

All compartments are also affected by a natural mortality rate μ.

Differential Equations of the Model

$$\frac{dS}{dt} = \pi - \lambda S - \mu S$$

$$\frac{dD_1}{dt} = p\lambda S - (a + \sigma_1 + \mu)D_1$$

$$\frac{dD_2}{dt} = (1 - p)\lambda S - (\sigma_2 + \mu)D_2$$

$$\frac{dA}{dt} = \sigma_1 D_1 + \sigma_2 D_2 - (\delta_1 + \gamma + \mu)A$$

$$\frac{dR_1}{dt} = \gamma A - (\tau + \delta_2 + \mu)R_1$$

$$\frac{dR_2}{dt} = aD_1 + \tau R_1 - \mu R_2$$

Where:

$$\lambda = \frac{\beta A S}{N}, \quad N(t) = S + D_1 + D_2 + A + R_1 + R_2$$

Parameter Definitions

Symbol	Description
π	Recruitment rate of susceptible individuals
p	Proportion transitioning to secret users
μ	Natural mortality rate
β	Effective contact rate
a	Awareness rate (contributing to recovery)
γ	Rehabilitation rate
τ	Recovery rate from rehabilitation
σ_1, σ_2	Progression rates to addiction from D_1 and D_2, respectively
δ_1, δ_2	Drug-induced mortality rates from A and R_1

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Positivity and Boundedness of the Model
Let

$$H = \{(S, D_1, D_2, A, R_1, R_2) \in \mathbb{R}^6_+ : N(t) \leq \frac{\pi}{\mu}\}$$

be the bio logically feasible reg ion.

Theorem 3.1: The regi"on H is posi"tively i"nvari"ont and attracts all positive solutions a(system (I).
Proof Sketch:
AII Compartment s a re shown to be non-negative for alI () using differential inequalities:
For example:

$$\frac{dS}{dt} \geq -(\lambda + \mu)S \Rightarrow S(t) \geq S(0)e^{-(\lambda+\mu)t}$$

Similar baunds are derived far D , D2, . ‹ . '
The total population satisfies

$$\frac{dN}{dt} = \pi - \mu N - \delta_1 A - \delta_2 R_1 \leq \pi - \mu N$$

Solving the inequality yields:

$$N(t) \leq \frac{\pi}{\mu} + \left(N(0) - \frac{\pi}{\mu}\right) e^{-\mu t}$$

Thus, the solutions remain within a bounded region II, confirming positive invariante.

Drug- Free Equilibrium Point
In the a bsence af drug influence (i.e.,λ=0), the system stabil izes at the d rug-free equilib rium point:

$$\varepsilon_0 = \left(\frac{\pi}{\mu}, 0, 0, 0, 0, 0\right)$$

Basic Reproduction Number (R_0)

Using the next-generation matrix approach (Diekmann et al., 1990; 2010), we define:

Let **F** be the matrix of new infections and **V** be the matrix of transition terms:

$$F = \begin{bmatrix} \frac{p\beta AS}{N} \\ \frac{(1-p)\beta AS}{N} \\ 0 \\ 0 \end{bmatrix}, \quad V = \begin{bmatrix} k_1 D_1 \\ k_2 D_2 \\ -\sigma_1 D_1 - \sigma_2 D_2 + k_3 A \\ -\gamma A + k_4 R_1 \end{bmatrix}$$

Linearizing at the drug-free equilibrium gives matrices for evaluating:

$$R_0 = \rho(FV^{-1})$$

II. CONCLUSION

In this study, a deterministic compartmental model was developed to explore the dynamics of drug addiction within a population segmented into six distinct classes: susceptible individuals, secret drug users, public drug users , addicted individuals , those undergoing rehabilitation , and fully recovered individuals. The model incorporates key real-world mechanisms, including initiation into drug use, progression to addiction, public vs. secret usage, rehabilitation, recovery, awareness, and drug-induced mortality.

Our analysis confirms that the model is biologically well-posed, as all its solutions remain positive and bounded within a feasible region defined by , ensuring the population remains finite and realistic over time. We have shown that this feasible region is positively invariant and globally attractive, meaning any trajectory beginning within it remains valid and converges over time.

The drug-free equilibrium point was identified and analyzed, representing a state where drug use is completely eradicated. Furthermore, we computed the basic reproduction number using the next-generation matrix approach, which serves as a threshold parameter indicating whether drug addiction will die out .The model provides critical insight into the influence of key parameters such as:

the awareness rate ( ), which effectively reduces secret usage and increases recovery, the rehabilitation rate ( ), essential for reducing the number of addicts, the progression rates (σ_1, σ_2), which drive the transition from use to addiction,

and the drug-induced mortality rates (δ_1, δ_2), which highlight the public health consequences.

Overall, the model underscores the importance of targeted awareness campaigns, early interventions, and sustained rehabilitation programs in curbing the drug epidemic. Policymakers can use such models to simulate various intervention strategies and forecast long-term outcomes, thereby enabling data-driven decisions for effective public health planning. Future work may incorporate stochastic effects, age-structured populations, or spatial dynamics to further enrich the model's applicability and realism.

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