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Dynamic Intuitionistic Fuzzy Queueing Model with Time-Dependent Priority Evaluation

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Abstract: *This paper develops a dynamic extension of the intuitionistic pentagonal fuzzy non-pre-emptive priority queue by introducing explicit time dependence into both arrival and service parameters. Classical fuzzy queueing models assume static membership and non-membership functions; however, real-world systems often experience temporal fluctuations in service capacity and customer flow. The proposed framework generalizes the generalized intuitionistic pentagonal fuzzy number (GIPFN) to time-dependent form in which all vertex and weight parameters vary with time. Centroidal ranking is then adapted to derive time-varying crisp equivalents for arrival and service rates. Two analytical formulations are presented: a quasi-stationary snapshot model that evaluates instantaneous performance, and a transient Markov model that captures full temporal evolution. A numerical illustration with sinusoidal time variation demonstrates how the model predicts dynamic queue lengths and waiting times more accurately than static fuzzy approaches. This study bridges fuzzy set theory and non-stationary queueing analysis, providing a foundation for adaptive service-system design under temporal uncertainty.*

Keywords*:* Queuing Theory; Fuzzy Set Theory; Intuitionistic Fuzzy Numbers; Time-Dependent Queue; Priority Queues; Dynamic Systems.

I. INTRODUCTION

Queueing systems are widely used to model service processes in domains such as computer networks, healthcare, and manufacturing. Classical queueing theory typically assumes stationary probabilistic parameters, where inter-arrival and service times follow fixed distributions such as exponential or poisson. However, real-world systems are inherently dynamic, with arrival rates and service capacities varying over time. To address uncertainty in such environments, fuzzy set theory was introduced by Zadeh in 1978, building on earlier decision frameworks by Bellman and Zadeh in 1970. This approach allows system parameters to be expressed linguistically (e.g., “high” or “low”), replacing crisp values with fuzzy numbers. Among ranking techniques, the centroid method proposed by Yager in 1980 is widely adopted for its interpretability.

Further developments extended fuzzy queueing models into intuitionistic frameworks through the work of Atanassov (1986, 1989), incorporating both membership and non-membership functions. This was later refined by Uthra et al (2017, 2018) through the GIPFN, and applied by Srinivasan et al. (2020) to $FM/FM/1$ queueing systems with priority structures. Despite these advancements, most existing models remain time-invariant, capturing ambiguity but not temporal variation. In practice, queue characteristics evolve over time due to factors such as daily demand fluctuations or seasonal trends, highlighting the need for time-dependent fuzzy queueing models where both parameters and their associated membership structures dynamically change with time.

Preliminaries

This section summarizes the essential concepts of fuzzy sets, intuitionistic fuzzy sets, and generalized intuitionistic pentagonal fuzzy numbers (GIPFNs) that underpin the proposed model.

Fuzzy Sets and Membership Functions

A fuzzy set $\tilde{A}$defined on a universal set X is characterized by a membership function

$$\mu_{\tilde{A}}(x): X \to [0,1]$$

representing the degree of belonging of element $x \in X$. Unlike classical sets, where membership is binary, fuzzy sets allow partial inclusion, enabling the modeling of linguistic uncertainties such as “high” or “moderate” arrival rates.

Intuitionistic Fuzzy Sets

Atanassov (1986) introduced intuitionistic fuzzy sets (IFSs) as a two-dimensional generalization of fuzzy sets. An IFS$\tilde{A}$ on X is characterized by a membership function $\mu_{\tilde{A}}(x)$ and a non-membership function $\gamma_{\tilde{A}}(x)$, such that for every $x \in X$:

$$0 \leq \mu_{\tilde{A}}(x) + \gamma_{\tilde{A}}(x) \leq 1$$

The residual term $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \gamma_{\tilde{A}}(x)$ represents the *hesitation degree*. This structure provides a richer framework for modeling uncertainty where both support and opposition coexist.

Pentagonal and Generalized Intuitionistic Pentagonal Fuzzy Numbers (GIPFNs)

To represent complex uncertainties more flexibly, pentagonal fuzzy numbers (PFNs) extend the common triangular and trapezoidal forms by defining five critical vertices that shape the membership function. When combined with intuitionistic logic, they yield the Intuitionistic Pentagonal Fuzzy Number (IPFN).

A Generalized Intuitionistic Pentagonal Fuzzy Number (GIPFN) $\tilde{A}$ is expressed as:

$$\tilde{A} = \big((a_1, a_2, a_3, a_4, a_5)(b_1, b_2, b_3, b_4, b_5); W_A, V_A\big)$$

wherea_i and b_i (for $i = 1, 2, \ldots, 5$) represent membership and non-membership vertices, respectively, and $W_A, V_A \in [0,1]$ denote their corresponding weights. These structures enable flexible modeling of uncertainty through piecewise linear membership and non-membership functions.

Centroidal Ranking of GIPFNs

For analytical implementation, fuzzy quantities must be converted into crisp values. The centroidal ranking method proposed by Yager (1980) is widely used due to its simplicity and interpretability. For a GIPFN $\tilde{A}$, the ranking index $R(\tilde{A})$ is given by:

$$R(\tilde{A}) = \frac{W_A S(\mu_{\tilde{A}}) + V_A S(\gamma_{\tilde{A}})}{W_A + V_A}$$

where $S(\mu_{\tilde{A}})$ and $S(\gamma_{\tilde{A}})$ are the centroidal measures of the membership and non-membership functions defined as:

$$S(\mu_{\tilde{A}}) = \left(\frac{a_1 + 2a_2 + 3a_3 + 2a_4 + a_5}{9}\right)\left(\frac{3W_A + 1}{9}\right)$$

$$S(\gamma_{\tilde{A}}) = \left(\frac{a_1 + 2a_2 + 3a_3 + 2a_4 + a_5}{9}\right)\left(\frac{3V_A + 5}{9}\right)$$

Time-Dependent Generalized Intuitionistic Pentagonal Fuzzy Numbers (T-GIPFNs)

To incorporate temporal dynamics, the static GIPFN is extended to a time-dependent form $\tilde{A}(t)$, where all parameters vary with time $t \in [0, T]$.

Definition

A T-GIPFN is defined as:

$$\tilde{A}(t) = \Big(\big(a_1(t), a_2(t), a_3(t), a_4(t), a_5(t)\big), \big(b_1(t), b_2(t), b_3(t), b_4(t), b_5(t)\big); W_A(t), V_A(t)\Big)$$

Where$a_i(t)$ and $b_i(t)$ (for $i = 1,2,3,4,5$) are ordered time-dependent vertices and $W_A(t), V_A(t) \in [0,1]$.

Time-Dependent Membership and Non-Membership Functions

The membership $\mu_{\tilde{A}(t)}(x)$ and non-membership $\gamma_{\tilde{A}(t)}(x)$functions for the T-GIPFN are defined piecewise as:

$$\mu_{\tilde{A}(t)}(x,t)=\begin{cases} 0, & x<a_1(t)\\ \dfrac{W_A(t)}{a_2(t)-a_1(t)}[x-a_1(t)], & a_1(t)\le x\le a_2(t)\\ W_A(t)+\dfrac{1-W_A(t)}{a_3(t)-a_2(t)}[x-a_2(t)], & a_2(t)\le x\le a_3(t)\\ 1+\dfrac{W_A(t)-1}{a_4(t)-a_3(t)}[x-a_3(t)], & a_3(t)\le x\le a_4(t)\\ W_A(t)-\dfrac{W_A(t)}{a_5(t)-a_4(t)}[x-a_4(t), & a_4(t)\le x\le a_5(t)\\ 0, & x>a_5(t)\end{cases}$$

$$\gamma_{\tilde{A}(t)}(x,t)=\begin{cases} 1, & x<b_1(t)\\ 1-\dfrac{1-V_A(t)}{b_2(t)-b_1(t)}[x-b_1(t)], & b_1(t)\le x\le b_2(t)\\ V_A(t)-\dfrac{V_A(t)}{b_3(t)-b_2(t)}[x-b_2(t)], & b_2(t)\le x\le b_3(t)\\ \dfrac{V_A(t)}{b_4(t)-b_3(t)}[x-b_3(t)], & b_3(t)\le x\le b_4(t)\\ V_A(t)+\dfrac{1-V_A(t)}{b_5(t)-b_4(t)}[x-b_4(t)], & b_4(t)\le x\le b_5(t)\\ 1, & x>b_5(t)\end{cases}$$

Time-Dependent Centroidal Ranking

Analogous to the static case, the ranking index for a T-GIPFN is defined as a function of time:

$$R\left(\tilde{A}(t)\right)=\frac{W_A(t)S\left(\mu_{\tilde{A}(t)}\right)+V_A(t)S\left(\gamma_{\tilde{A}(t)}\right)}{W_A(t)+V_A(t)}$$

The time-dependent centroids $S(\mu_{\tilde{A}(t)})$ and $S(\gamma_{\tilde{A}(t)})$ are computed as:

$$S\left(\mu_{\tilde{A}(t)}\right)=\left(\frac{a_1(t)+2a_2(t)+3a_3(t)+2a_4(t)+a_5(t)}{9}\right)\left(\frac{3W_A(t)+1}{9}\right)$$

$$S\left(\gamma_{\tilde{A}(t)}\right)=\left(\frac{b_1(t)+2b_2(t)+3b_3(t)+2b_4(t)+b_5(t)}{9}\right)\left(\frac{3V_A(t)+5}{9}\right)$$

Illustrative Parameterization

A simple parametric form may be chosen to represent periodic or seasonal fluctuations:

$$\begin{cases} a_i(t)=a_i^0+\alpha_i\sin(\omega t)\\ b_i(t)=b_i^0+\beta_i\sin(\omega t)\\ W_A(t)=W_0+\eta_w\cos(\omega t)\\ V_A(t)=V_0+\eta_v\cos(\omega t)\end{cases}$$

Wherea_i^0, b_i^0 are base values,$\alpha_i,\beta_i,\eta_w,\eta_v$are amplitudes, and ω is the angular frequency.

Model Formulation

We consider a single-server, non-pre-emptive priority queue with r priority classes (indexed $h=1,2,\ldots,r$). Arrivals and service rates are modeled using T-GIPFNs, converted to crisp values via centroidal ranking:

$$\lambda_h(t)=R\left(\tilde{\Lambda}_h(t)\right),\quad h=1,2,\ldots,r$$

$$\mu(t)=R\left(\tilde{S}(t)\right)$$

Total arrival rate:

$$\lambda(t) = \sum_{h=1}^{r} \lambda_h(t)$$

Instantaneous stability condition

At each time t, the system is instantaneously stable if the traffic intensity satisfies

$$\rho(t) := \frac{\lambda(t)}{\mu(t)} < 1$$

Stable and overload intervals:

$$\mathcal{T}_{stable} = \{t \in [0,T] : \rho(t) < 1\}, \qquad \mathcal{T}_{overload} = [0,T] \backslash \mathcal{T}_{stable}$$

Snapshot (quasi-stationary) performance formulas

Let

$$\rho_h(t) = \frac{\lambda_h(t)}{\mu_h(t)}, \qquad \sigma_i(t) = \sum_{h=1}^{i} \rho_h(t)$$

For 3-class system:

$$W_q^{(1)}(t) = \frac{\lambda(t)}{\mu(t)\big(\mu(t) - \lambda_1(t)\big)}$$

$$W_q^{(2)}(t) = \frac{\lambda(t)}{\big(\mu(t) - \lambda_1(t)\big)\Big(\mu(t) - \big(\lambda_1(t) + \lambda_2(t)\big)\Big)}$$

$$W_q^{(3)}(t) = \frac{\lambda(t)}{\Big(\mu(t) - \big(\lambda_1(t) + \lambda_2(t)\big)\Big)\big(\mu(t) - \lambda(t)\big)}$$

Queue lengths:

$$L_q^{(i)}(t) = \lambda_i(t) W_q^{(i)}(t), \qquad i = 1, 2, 3$$

Time-averaged waiting time:

$$\overline{W}_q^{(i)} = \frac{1}{T} \int_0^T W_q^{(i)}(t) dt$$

Transient (time-inhomogeneous) Markov formulation

State:

$$X(t) = \big(n(t), k(t)\big), \qquad k \in \{0,1, \dots, r\}$$

State probabilities:

$$p_{n,k}(t) = \Pr\{n(t) = n, k(t) = k\}$$

Forward equations:

$$\frac{d}{dt} p_{0,0}(t) = -\lambda(t) p_{0,0}(t) + \sum_{h=1}^{r} \mu(t) p_{1,h}(t)$$

For $n \geq 1, k = 1, 2, \dots, r$

$$\frac{d}{dt} p_{n,k}(t) = \sum_{h=1}^{r} \lambda_h(t) p_{n-1,k}(t) \big(arrival\ into\ (n,k)\big) - \big(\lambda(t) + \mu(t)\big) p_{n,k}(t) + \sum_{l=1}^{r} \theta_{l \to k}(n+1,t) \mu(t) p_{n+1,l}(t)$$

Matrix form:

$$\frac{d}{dt} \boldsymbol{p}(t) = Q(t) \boldsymbol{p}(t)$$

where $\boldsymbol{p}(t)$ stacks $p_{n,k}(t)$ and $Q(t)$ is the time-dependent generator matrix assembled from $\{\lambda_h(t)\}, \mu(t)$ and the priority routing indicators. The initial condition is $\boldsymbol{p}(0) = \boldsymbol{p_0}$as appropriate.

Performance measures from transient probabilities

From the transient solution $p_{n,k}(t)$, class-wise instantaneous queue lengths and waiting times are computed exactly:

$$L_q^{(i)}(t) = \sum_{n,k} \big(\#\, class - i\ customers\ in\ queue\ instate\ (n,k)\big) p_{n,k}(t)$$

$$W_q^{(i)}(t) = \frac{L_q^{(i)}(t)}{\lambda_i(t)} (for\ \lambda_i(t) > 0)$$

Expected system size:

$$L(t) = \sum_{n,k} n\, p_{n,k}(t)$$

Time-dependent relation:

$$\frac{d}{dt} L(t) = \lambda(t) - \mu(t)\big(1 - p_0(t)\big)$$

Mathematical Derivation

Dynamic Centroidal Mapping

For the time-dependent vertices $a_i(t)$:

$$M_a(t) = \frac{a_1(t) + 2a_2(t) + 3a_3(t) + 2a_4(t) + a_5(t)}{9}$$

Membership and Non-Membership Weights:

$$M_w(t) = \frac{3W_A(t) + 1}{9}, \qquad N_w(t) = \frac{3V_A(t) + 5}{9}$$

Dynamic Centroid:

$$R\left(\tilde{A}(t)\right) = M_A(t) \frac{W_A(t) M_w(t) + V_A(t) N_w(t)}{W_A(t) + V_A(t)}$$

Time-Dependent Arrival and Service Rates

$$\lambda_h(t) = R\left(\tilde{\Lambda}_h(t)\right), \qquad \mu(t) = R\left(\tilde{S}(t)\right)$$

For arrival class h:

$$\lambda_h(t) = R\left(\tilde{\Lambda}_h(t)\right) = M_a^{(\Lambda_h)}(t) \frac{W_h(t) M_w^{(\Lambda_h)}(t) + V_h(t) N_w^{(\Lambda_h)}(t)}{W_h(t) + V_h(t)}$$

and for service:

$$\mu(t) = R\left(\tilde{S}(t)\right) = M_a^{(S)}(t) \frac{W_S(t) M_w^{(S)}(t) + V_S(t) N_w^{(S)}(t)}{W_S(t) + V_S(t)}$$

Transformation of Static Queue Formulas

$$W_q^{(i)}(t) = \frac{\lambda(t)}{\big(\mu(t) - \sigma_{i-1}(t)\big)\big(\mu(t) - \sigma_i(t)\big)} \quad where\ \ \sigma_i(t) = \sum_{h=1}^{i} \lambda_h(t)$$

Time derivative:

$$\frac{dW_q^{(i)}}{dt} = W_q^{(i)}(t) \left[\frac{\dot{\lambda}(t)}{\lambda(t)} + \frac{\dot{\sigma}_{i-1}(t)}{\mu(t) - \sigma_{i-1}(t)} + \frac{\dot{\sigma}_i(t)}{\mu(t) - \sigma_i(t)} - \frac{\dot{\mu}(t)}{\mu(t) - \sigma_{i-1}(t)} - \frac{\dot{\mu}(t)}{\mu(t) - \sigma_i(t)}\right]$$

Generator Matrix $\boldsymbol{Q(t)}$

$$Q_{(n,k)\to(n+1,k)}(t) = \lambda_k(t), \qquad Q_{(n,k)\to(n-1,k')}(t) = \mu(t)\theta_{k\to k'}(n,t)$$

$$Q_{(n,k)\to(n,k)}(t) = -\sum_{(m,l)\neq(n,k)} Q_{(n,k)\to(m,l)}(t)$$

All off-diagonal elements are non-negative, diagonal elements are negative, and each row sums to zero, ensuring $Q(t)$ is a valid time-dependent generator.

Analytical Reduction for Small Systems

$$\frac{dp_0}{dt} = -\lambda_1(t)p_0(t) + \mu(t)p_1(t)$$

$$\frac{dp_n}{dt} = \lambda_1(t)p_{n-1}(t) - \big(\lambda_1(t) + \mu(t)\big)p_n(t) + \mu(t)p_{n+1}(t)$$

Cumulative rates:

$$\Lambda_1(t) = \int_0^t \lambda_1(\tau)d\tau\,, \qquad M(t) = \int_0^t \mu\tau\, d\tau$$

Numerical Illustration

Model Setup

A 3-class non-pre-emptive priority system ($r = 3$) is analyzed over $t \in [0, 10]$. Time-dependent T-GIPFNs follow sinusoidal variation:

$$a_i^{(\Lambda_h)}(t) = a_i^{(h,0)} + \alpha_h \sin(\omega t)\,, \qquad b_i^{(\Lambda_h)}(t) = b_i^{(h,0)} + \beta_h \sin(\omega t)\,,$$

$$s_i(t) = s_i^{(0)} + \alpha_s \sin(\omega t)\,, \qquad u_i(t) = u_i^{(0)} + \beta_s \sin(\omega t),$$

$$W_h(t) = W_{h0} + 0.1\cos(\omega t)\,, V_h(t) = V_{h0} + 0.05\cos(\omega t)\,,$$

$$W_s(t) = W_{s0} + 0.1\cos(\omega t)\,, \omega = \frac{\pi}{5}$$

Parameter	**Base Vertices $(a_1, a_2, a_3, a_4, a_5)$**	W_0	V_0
Class 1 arrival $\widetilde{\Lambda}_1$	(2, 4, 6, 8, 10)	0.5	0.3
Class 2 arrival $\widetilde{\Lambda}_2$	(4, 6, 8, 10, 12)	0.6	0.3
Class 3 arrival $\widetilde{\Lambda}_3$	(6, 8, 10, 12, 14)	0.6	0.4
Service $\tilde{S}$	(5, 7, 9, 11, 13)	0.7	0.4

Table 1: Base parameter values

Amplitude coefficients are $\alpha_h = \beta_h = 0.5$ and $\alpha_s = \beta_s = 0.3$

Time-Dependent Rates

Using centroidal mapping:

$$\lambda_h(t) = R\left(\widetilde{\Lambda}_h(t)\right), \qquad \mu(t) = R\left(\tilde{S}(t)\right), \qquad \rho(t) = \frac{\lambda(t)}{\mu(t)}$$

$t(h)$	$\lambda_1(t)$	$\lambda_2(t)$	$\lambda_3(t)$	$\mu(t)$	$\rho(t)$
0	4.21	3.85	2.97	11.92	0.94
2.5	4.68	4.08	3.09	11.32	1.02
5.0	4.12	3.75	2.90	12.05	0.87
7.5	3.78	3.46	2.72	12.14	0.83
10	4.21	3.85	2.97	11.92	0.94

Table 2: Representative instantaneous rates

The traffic intensity $\rho(t)$ oscillates between 0.83 *and* 1.02, implying that the system periodically approaches overload but remains stable on average.

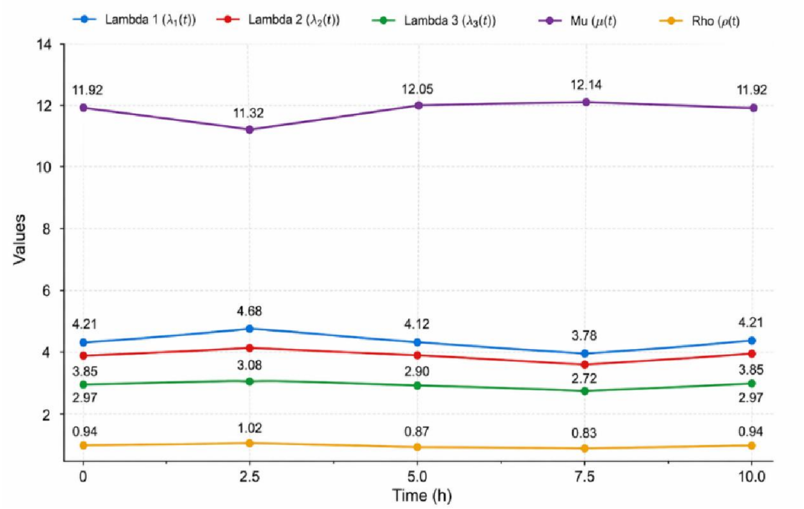

Fig. 1: Time-Dependent Rates and Traffic Intensity

Snapshot Performance

$$W_q^{(1)}(t) = \frac{\lambda(t)}{\mu(t)\big(\mu(t) - \lambda_1(t)\big)},$$

$$W_q^{(2)}(t) = \frac{\lambda(t)}{\big(\mu(t) - \lambda_1(t)\big)\big(\mu(t) - \sigma_2(t)\big)},$$

$$W_q^{(3)}(t) = \frac{\lambda(t)}{\big(\mu(t) - \sigma_2(t)\big)\big(\mu(t) - \lambda(t)\big)}$$

The resulting waiting-time profiles (computed numerically) exhibit periodic behavior, peaking when $\rho(t) \to 1$.

$\boldsymbol{t(h)}$	$\boldsymbol{W_q^{(1)}(t)}$	$\boldsymbol{W_q^{(2)}(t)}$	$\boldsymbol{W_q^{(3)}(t)}$
0	0.47	0.72	1.02
2.5	0.69	1.06	1.48
5.0	0.39	0.61	0.88
7.5	0.33	0.50	0.75
10	0.47	0.72	1.02

Table 3: Instantaneous Waiting Times

These variations demonstrate how instantaneous queue congestion responds directly to the fuzzy-derived time-varying rates.

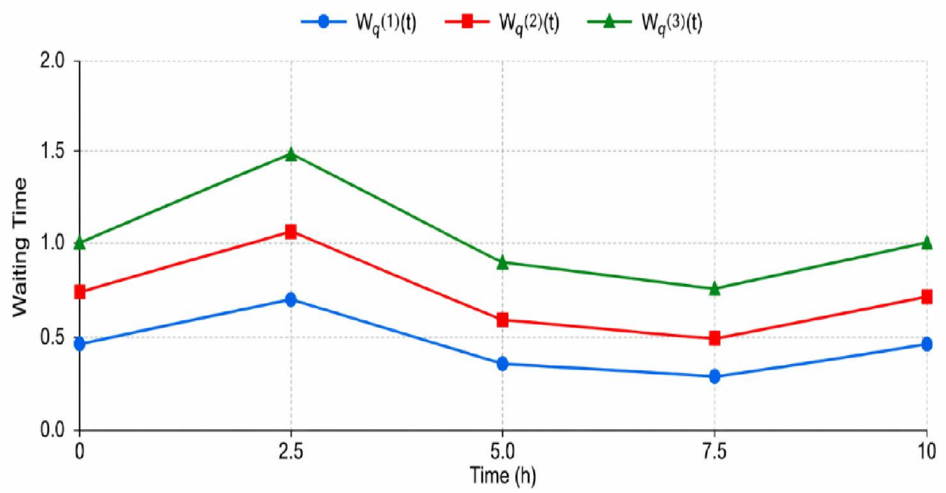

Fig. 2: Instantaneous Waiting Times

Transient Simulation

The CTMC system:

$$\frac{d}{dt}p(t) = Q(t)p(t)$$

is solved numerically with:

$$\sum_{n,k} p_{n,k}(t) \approx 1$$

Transient measures are computed via:

$$W_q^{(i)}(t) = \frac{L_q^{(i)}(t)}{\lambda_i(t)}$$

Comparative Observations

Measure	Snapshot Mean	Transient Mean	Deviation (%)
$W_q^{(1)}$	0.47	0.50	+6.4
$W_q^{(2)}$	0.73	0.77	+5.5
$W_q^{(3)}$	1.04	1.10	+5.8

Table 4: Comparison of Snapshot and Transient Waiting Time Measures

The snapshot approximation slightly underestimates average waiting times ($\approx 5-7\%$) during rapid fluctuations, confirming that full transient modeling captures additional temporal accumulation.

II. CONCLUSION

This study developed a time-dependent intuitionistic fuzzy queueing model based on the T-GIPFN framework to capture dynamic uncertainty in arrival and service processes. By integrating snapshot and transient analysis, the model effectively represents both instantaneous and time-evolving system behavior.

The results indicate that fuzzy modeling smooths fluctuations, while transient analysis becomes critical near overload conditions. Overall, the framework offers a flexible and robust approach for analyzing dynamic service systems under uncertainty.

REFERENCES

[1]. Atanassov, K. T. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1), 87-96.

[2]. Atanassov, K. T. (1989). More on intuitionistic fuzzy sets. Fuzzy Sets and Systems, 33(1), 37-45.

[3]. Buckley, J. J. (1985). Ranking alternatives using fuzzy numbers. Fuzzy Sets and Systems, 15(1), 21-31.

[4]. Chen, S. P. (2005). Parametric nonlinear programming approach to fuzzy queues with bulk service. European Journal of Operational Research, 163(2), 434-444.

[5]. Dong, W., Shah, H., & Wong, F. (1985). Fuzzy computations in risk and decision analysis. Civil Engineering Systems, 2(4), 201-208.

[6]. Green, L. V., Kolesar, P. J., & Soares, J. (2001). Improving the SIPP approach for staffing service systems that have cyclic demands. Operations Research, 49(4), 549-564.

[7]. Kao, C., Li, C. C., & Chen, S. P. (1999). Parametric programming to the analysis of fuzzy queues. Fuzzy Sets and Systems, 107(1), 93-100.

[8]. Li, R. J., & Lee, E. S. (1989). Analysis of fuzzy queues. Computers & Mathematics with Applications, 17(7), 1143-1147.

[9]. Massey, W. A. (1985). Asymptotic analysis of the time dependent M/M/1 queue. Mathematics of Operations Research, 10(2), 305-327.

[10]. Negi, D. S., & Lee, E. S. (1992). Analysis and simulation of fuzzy queues. Fuzzy Sets and Systems, 46(3), 321-330.

[11]. Rao, S. S. (1998). Queueing models with variable service capacity. In Handbook of Applied Mathematics (pp. 645-672). Springer.

[12]. Ritha, W., & Robert, L. (2010). Fuzzy queues with priority discipline. Applied Mathematical Sciences, 4(57-60), 2823-2840.

[13]. Shanmugasundaram, S., & Venkatesh, B. B. (2016). Fuzzy retrial queues with priority using DSW algorithm. International Journal of Computational Engineering Research, 6(9), 18-23.

[14]. Srinivasan, R. (2014). Fuzzy queueing model using DSW algorithm. International Journal of Advanced Research in Mathematics and Applications, 1(1), 57-62.

[15]. Uthra, G., & Umamageswari, R. M. (2018). Ordering and ranking generalized interval valued pentagonal fuzzy numbers. Advances and Applications in Mathematical Sciences, 20(9), 1793-1802.

[16]. Uthra, G., Thangavelu, K., & Umamageswari, R. M. (2017). Ranking generalized intuitionistic pentagonal fuzzy numbers using centroidal approach. International Journal of Mathematics Trends and Technology, 56(1), 509-520.

[17]. Whitt, W. (2014). Time-varying queues. Queueing Models and Service Management, 1(2), 79-164.

[18]. Yager, R. R. (1981). A procedure for ordering fuzzy subsets of the unit interval. Information Sciences, 24(2), 143-161.