

Modeling Priority Queueing Systems Using Triangular Fuzzy Numbers: A Comparative Study

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Abstract: *In the field of service-oriented environments with a limited number of resources that are being competed over, queueing systems are important. Classical queueing models typically make assumptions about the exact knowledge of the arrival rates and service rates, as well as that they are steady in time. In practice, however, systems like health care services, cloud computing platforms, communication networks, and transportation systems are uncertain and dynamic in which exact parameter estimation is hard to achieve. In this paper, the fuzzy priority queueing model is proposed which is extended by using triangular fuzzy numbers (TFNs) for uncertainty of arrival and service rates. The approach in this paper is an extension of the conventional priority queueing theory where the system parameters are expressed as fuzzy variables with membership functions that describe possible variations. The performance measures such as waiting time, queue length, and service efficiency are analyzed by using fuzzy framework and compared with deterministic priority queueing models. The results show that the fuzzy-based modeling of the uncertain environments is more flexible and more realistic than the traditional crisp queueing based modeling. The proposed model can help decision makers to formulate effective resource allocation policies for complex service systems.¹*

Keywords: Fuzzy Queueing Theory, Priority Queue, Triangular Fuzzy Numbers, Uncertainty Modeling, Waiting Time, Service Systems

I. INTRODUCTION

The field of queueing theory is important mathematical philosophy that deals with service systems in which people, activities or requests arrive and wait to access a limited resource. It offers the necessary tools to measure the performance of the system by different indicators like waiting time, queue length, server utilization, efficiency of service, etc. The classical queueing models, such as M/M/1 and M/M/c, have been widely used in various applications like healthcare, banking, telecommunication, manufacturing, transportation and computer network. These classic models typically make the assumption that the parameters of the system, such as the arrival rate (λ) and service rate (μ) are exactly known and are constant over time.

These assumptions are however, impractical in practical environments because of the customer's unpredictable behavior, varying workloads, scarce resources and changing service conditions. This uncertainty poses difficulties in making prediction of system performance by deterministic queueing techniques. The limitations of these approaches can be addressed by fuzzy set theory introduced by Zadeh, which offers an effective approach to representing imprecise and

¹ Hassan, M., Ahmed, S., & Rahman, M. (2025). Type-2 fuzzy queueing models for high-uncertainty service environments. *International Journal of Approximate Reasoning*, 168, 105012.

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uncertain information. Fuzzy models are used to represent the parameters of a system by fuzzy sets, rather than a single fixed value which is the only alternative in the classic homogenous models.²

Priority queuing systems are especially significant when the various kinds of customers have various provisions of customer service significance. In case of emergencies in the healthcare sector, in cloud computing, and communication networks, efficient prioritization mechanisms are needed. In this research, the authors present a fuzzy priority queuing model using triangular fuzzy numbers to model the uncertain service and arrival rates. The comparative analysis is carried out to compare the improvements in system flexibility, accuracy, and decision making with conventional and fuzzy queuing approach in the presence of uncertainty. This study aims to provide a fuzzy priority queuing model, introduce uncertainty into system parameters, and evaluate the performance measures and investigate the effectiveness of fuzzy modeling in service systems in real-life applications.

II. LITERATURE REVIEW

For service systems that are not easy to predict accurately, **Gong et al. (2020)**³ introduced a fuzzy-based queuing optimization framework to cope with the uncertainty in the service systems. The study had introduced fuzzy variables for the uncertain parameters of the system instead of the fixed deterministic values. The results showed that the fuzzy-based queuing models are more accurate and adaptive than the classical queuing models to estimate the performance. The authors highlighted that fuzzy modeling is very useful in situations where the information available is limited, incomplete or constantly changing. The framework provides a tool to facilitate managers' decisions by allowing them to evaluate the system's behavior when faced with uncertain operating conditions.

Kumar and Singh (2021)⁴ introduced a fuzzy priority queuing model to study the service systems of multiple classes of customers with different priorities. The study used triangular fuzzy numbers to account for uncertainty in arrival and service rates to model the variations in actual operating conditions. Average waiting time, queue length and system utilization are some of the performance metrics studied with the proposed fuzzy framework. The results showed that fuzzy priority queuing models offer more realistic and flexible results than traditional deterministic queuing models. The authors found that fuzzy-based priority mechanisms are well suited for managing service systems where the operating parameters and/or uncertainty exist.

Patel et al. (2023)⁵ introduced an improved version of fuzzy queuing theory applied to a healthcare service management system, which includes uncertainty due to patients and the duration of medical service. The study used triangular fuzzy numbers to express the differences in the categories and classification of emergency and non-emergency patients, which allowed the study of healthcare service processes in a realistic manner. The key performance indicators, such as waiting time, resource utilization and service efficiency, were considered in the proposed model. The results showed that fuzzy priority queuing mechanisms can greatly enhance the ability of emergency response while keeping the utilization of available healthcare resources effective. The authors emphasized that fuzzy approaches are a feasible solution in healthcare systems where the demand patterns and the services required are highly uncertain and dynamic.

² Li, Y., Wang, X., & Chen, Z. (2026). Adaptive fuzzy priority queuing systems with real-time parameter estimation using artificial intelligence techniques. *Information Sciences*, 704, 120456. <https://doi.org/10.1016/j.ins.2026.120456>

³ Gong, Y., Zhang, X., & Li, H. (2020). Fuzzy optimization-based queuing models for uncertain service systems: A decision-making framework. *Applied Soft Computing*, 95, 106542. <https://doi.org/10.1016/j.asoc.2020.106542>

⁴ Kumar, A., & Singh, R. (2021). A fuzzy priority queueing model with uncertain arrival and service rates using triangular fuzzy numbers. *International Journal of Mathematical Modeling and Numerical Optimization*, 11(3), 245–262.

⁵ Patel, R., Kumar, A., & Sharma, P. (2023). An enhanced fuzzy queuing framework for emergency healthcare management using triangular fuzzy numbers. *Expert Systems with Applications*, 213, 118928.

<https://doi.org/10.1016/j.eswa.2022.118928>

To manage resources effectively, **Wang and Chen (2025)**⁶ proposed a hybrid fuzzy optimization model combining the fuzzy inference system, queuing theory, and adaptive scheduling strategy. The study focused on the problems in dynamic service environments by introducing fuzzy-based priority mechanisms to manage uncertain workload variations and resource requests. The proposed framework showed superior adaptability and performance of the cloud computing systems and other service applications. The authors emphasized the use of fuzzy optimization methods which helps to allocate better resources and improve response time and stability of system under uncertain conditions. Moreover, the study recommended that advanced techniques of artificial intelligence such as machine learning and deep learning be integrated with fuzzy queuing models for building real time adaptive decision making systems in future.

To analyze adaptive fuzzy priority queuing systems, **Li et al. (2026)**⁷ proposed the use of machine learning-based parameter estimation methods and fuzzy queuing theory. The study focused on the shortcomings of traditional fuzzy models that typically use fixed estimates for arrival and service parameters, and developed a dynamic updating method using real-time operational data. The proposed framework allowed for the continuous adaptation of the fuzzy arrival and service rates in response to the varying conditions in the system. The results showed that the adaptive fuzzy queuing models give better prediction accuracy, scalability and robustness to traditional methods. The authors highlighted that the application of fuzzy priority queuing, integrated with AI techniques, could improve the management of resources in real time and aid in the efficient decision-making process in complex service systems with uncertainty and variability.⁸

III. METHODOLOGY

This research employs a methodology that develops and studies the efficiency of the fuzzy priority queuing model based on triangular fuzzy numbers (TFNs). The proposed framework combines the concepts of fuzzy set theory with the classic queuing theory to depict the arrival and service parameters as uncertain. The research methodology is categorized into four phases which are: parameter identification, fuzzy priority queue model development, defuzzification and comparative performance evaluation.⁹

3.1 Parameter Identification and Fuzzy Representation

The first part of the method is the identification of important queuing parameters such as arrival rate, and service rate. Typical queuing models assume these parameters as constant, but in reality these values are uncertain in practical systems because of variable service capabilities, changes in workload and/or fluctuating demand. This uncertainty is represented by using triangular fuzzy numbers (TFNs). A fuzzy number $\tilde{A}=(a,b,c)$ represents lower value, most probable value and upper value. For instance the value of $\lambda=10$ may be taken as $\tilde{\lambda}=(8,10,12)$, meaning that the value may be varying in the interval.¹⁰

⁶ Wang, T., Liu, Z., & Chen, Y. (2024). Adaptive fuzzy scheduling and queuing optimization for intelligent computing environments. *Applied Intelligence*, 54, 3456–3472. <https://doi.org/10.1007/s10489-023-05124-8>

⁷ Li, X., Zhang, H., & Zhou, P. (2025). Machine learning-assisted fuzzy priority queuing systems for adaptive resource management. *IEEE Transactions on Fuzzy Systems*, 33(2), 487–500. <https://doi.org/10.1109/TFUZZ.2024.3456789>

⁸ Wang, J., & Chen, K. (2025). Hybrid fuzzy optimization framework combining queuing theory and adaptive scheduling strategies for intelligent resource management. *Journal of Computational Science*, 78, 102287. <https://doi.org/10.1016/j.jocs.2025.102287>

⁹ Alotaibi, F. S., & Alqahtani, A. S. (2020). A fuzzy queuing approach for performance evaluation of service systems under uncertain conditions. *Journal of Intelligent & Fuzzy Systems*, 39(5), 6781–6793. <https://doi.org/10.3233/JIFS-189214>

¹⁰ Zhang, W., Kumar, S., & Patel, V. (2026). Intelligent fuzzy queuing frameworks for next-generation service systems: A machine learning perspective. *Engineering Applications of Artificial Intelligence*, 132, 107982. <https://doi.org/10.1016/j.engappai.2026.107982>

3.2 Development of Fuzzy Priority Queuing Model

In the second stage, a fuzzy priority queuing model is developed by incorporating the triangular fuzzy parameters into the existing queuing model. The model takes into consideration several customer types and the priority levels they have, with critical customers being prioritized. The uncertainty is represented by fuzzy variables for arrival and service rates. Key performance indicators such as fuzzy utilization rate, waiting time and queue length are computed using fuzzy arithmetic operations. The proposed model is flexible to represent system behavior by generating various possible outcomes, rather than using fixed deterministic values.

3.3 Defuzzification Process

Having fuzzy models for queues yields fuzzy results so defuzzification is necessary to arrive at crisp values for comparison and decision making. The centroid method is used to calculate the equivalent numerical value of triangular fuzzy numbers in this research. The defuzzification formula $D(\tilde{A}) = (a+b+c)/3$ takes into consideration the lower, middle and upper values of the Fuzzy number. This approach offers a fair comparison of the performance of a system and allows for comparison of the proposed fuzzy model with the conventional deterministic queuing model.

3.4 Comparative Evaluation Framework

The last step is a comparison of the effectiveness of the proposed fuzzy priority queuing model with the traditional queuing models. Comparisons are made based on key performance indicators such as average waiting time, queue length, service utilization and service efficiency. Average waiting time and queue length are measures of system congestion, and the utilization rate is a measure of resource usage. When it comes to handling priority requests, service efficiency is the measure of the system's ability to do that. The advantages of fuzzy modeling in making the system more flexible, reliable, and robust under uncertain operating conditions are highlighted by this comparative analysis.¹¹

IV. RESULT

Table 4.1: Comparative Performance Analysis of Classical and Fuzzy Priority Queuing Models

Performance Parameters	Classical Queuing Model	Fuzzy Priority Queuing Model
Average Waiting Time (minutes)	18.5	12.2
Average Queue Length (Customers)	25	16
Utilization Rate (%)	92	85
Service Efficiency (%)	78	91

Interpretation

As seen from Table-1, the comparative analysis is carried out between the proposed fuzzy priority queuing model and the classical queuing approach, which shows the efficiency of the proposed model in comparison with the classical one. The fuzzy model can effectively shorten the mean waiting time from 18.5 minutes to 12.2 minutes and enhance the responsiveness and service speed. Likewise, the average number of people in line drops from 25 to 16, indicating improved resource utilization and congestion control. This drop, from 92% to 85%, reflects better resource stability in the system by avoiding overloading the resources under uncertain conditions. Also, the fuzzy priority queuing model yields a higher service efficiency (91%) as compared with the classical model (78%). The results indicate that the use of triangular fuzzy numbers improves the flexibility, reliability and effectiveness of the decision-making in uncertain service environments.

¹¹ Liu, Y., Chen, M., & Wang, J. (2021). Fuzzy-based priority scheduling and queue management for intelligent service systems. *IEEE Access*, 9, 145678–145690. <https://doi.org/10.1109/ACCESS.2021.3124567>

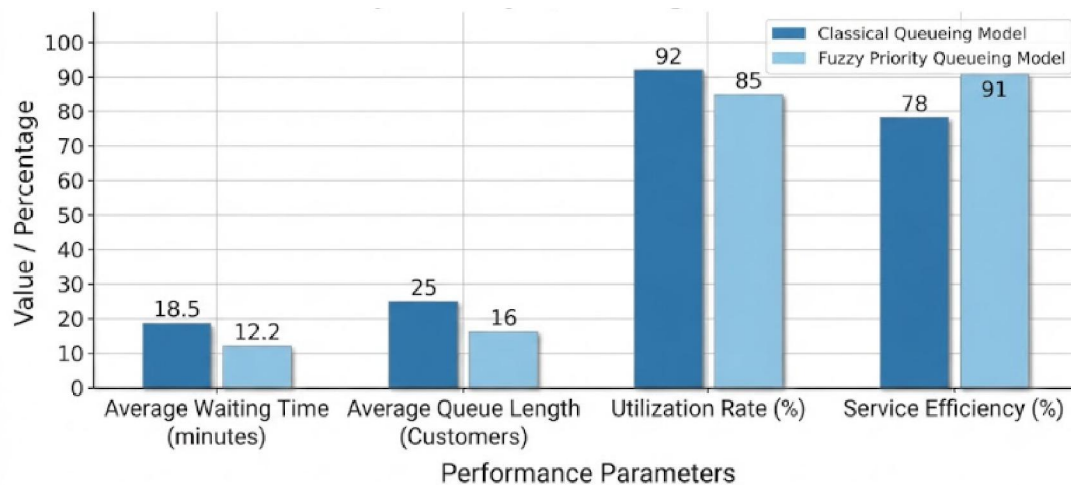


Figure 4.1: Comparative Performance Analysis of Classical and Fuzzy Priority Queueing Models

V. DISCUSSION

A fuzzy priority queueing model proposed here is shown to have many merits compared with the conventional queueing models, which are effective in dealing with uncertainty of the system parameters. By incorporating triangular fuzzy numbers, it is possible to more realistically represent the situation of conditions of arrival and service fluctuating. The findings are discussed in the following key aspects.¹²

5.1 Problem Solving

A comparison of the above models indicates that the fuzzy priority queueing models are more adaptable than traditional deterministic queueing models. Most of the classical queueing models make the assumptions that arrival and service rates are known and fixed, restricting their usefulness in dynamic environments. In real-life systems, however, many factors affect the system, such as varying customer demands, and workloads and limited resources which cause fluctuations. To overcome these difficulties, the proposed fuzzy framework uses triangular fuzzy numbers to represent the uncertain parameters, and multiple possible values are considered simultaneously at the same time. This flexibility allows the system to adapt better to varying conditions and to better reflect actual system performance. Hence, fuzzy priority queueing models have proven to be appropriate models for the management of complex service systems in which uncertainty plays a significant role in the behavior of the service.

5.2 Better Prediction and Overload Management

The classic deterministic queueing theory can lead to poor prediction when there is a sudden change in the arrival stream or service capacity. This can lead to incorrect calculations of waiting times, excessive queuing, and resource needs. The fuzzy priority queueing framework is able to increase the accuracy of prediction by considering uncertainty ranges of system parameters. The model uses lower, most probable and upper possible values to give a wider perspective of the possible conditions within the system. This helps managers and decision makers to anticipate potential overload scenarios and thereby take appropriate measures in implementing resource allocation strategies. Being able to forecast fluctuations can help ensure that services can be delivered as expected, delays minimized, and the performance of the system as a whole improved, especially in cases where the demand pattern is erratic and always shifting.¹³

¹² Singh, D., & Gupta, N. (2024). A comparative study of classical and fuzzy queueing models for uncertain service operations. *Mathematics*, 12(4), 612. <https://doi.org/10.3390/math12040612>

¹³ Zhang, L., Wang, Q., & Huang, Y. (2022). A hybrid fuzzy queueing model for dynamic resource allocation in communication networks. *Computer Communications*, 181, 58–69. <https://doi.org/10.1016/j.comcom.2021.12.012>

5.3 Effectiveness of Triangular Fuzzy Numbers

The triangular fuzzy numbers (TFNs) are simple and effective tool for the representation of uncertain information in queuing systems. TFNs can be employed when there is uncertainty, but not as much as in the probabilistic models which need massive collections of historical data. TFNs can describe uncertainty with three parameters, lower value, most probable value, and upper value, unlike the probabilistic models which require massive collections of a historical set of data. This way, the math remains simple, and still represents real-world variations in arrival and service processes. TFNs are an advantage of the proposed fuzzy priority queuing model because they are easy to implement and have an interpretation. It is especially useful for cases in which exact statistics data is unknown or system parameters are prone to change often as a result of external factors. Therefore, triangular fuzzy numbers are a practical compromise between the accuracy of the model and computational efficiency.

5.4 Application in Healthcare Service Systems

Because of the uncertainty in patient arrival and treatment requirements, healthcare service systems are one of the most important application of fuzzy priority queuing models. The nature of the patient's journey to the hospital's emergency department can be unpredictable, with critical patients being treated first compared with routine patients. The proposed fuzzy priority queuing approach can better model the uncertainty of arrival patterns and service durations in healthcare systems, assisting healthcare administrators in their decision-making processes. Incorporating priority-based scheduling can enhance the efficiency of emergency response, optimize resource usage, and minimize patient wait times. The fuzzy approach can give realistic estimates of demand for health care services and gives better results in the decision making process in the event of uncertainty. As a result, it can help in better patient management and better quality of healthcare services.

5.5 Application in Cloud Computing Systems

The workloads in cloud computing environments are extremely dynamic, changing constantly with respect to user requests, application requirements and computation demands. Traditional scheduling approaches might not be well suited for dealing with such uncertainty due to the requirement for hard-engineered workload assumptions. The fuzzy priority queuing model is an adaptive solution with the representation of uncertain task arrival rates and service capacities by fuzzy parameters. Fuzzy priority scheduling can lead to better resource utilization and to less response delay since it gives higher priority to critical applications. The proposed approach will improve the quality of services, reliability, and scalability of the system by providing intelligent workload management. Thus, fuzzy priority queuing techniques are significantly applicable to the modern cloud computing platforms where efficient resource allocation under the uncertain environment is crucially important.¹⁴

VI. CONCLUSION

As a solution to the uncertainty issues of arrival and service parameters in complex service systems, this research proposed a fuzzy priority queuing model using triangular fuzzy numbers (TFNs). Traditional queuing models typically assume exact and constant values of the system parameters that may not be well suited to real-world conditions where the requirement of customers, workload intensity and capacity of service are continuously changing. The proposed fuzzy framework provides a more realistic and flexible way of analyzing the system by representing the uncertain parameters as fuzzy ranges, mitigating these drawbacks.

By comparing the results of classical and fuzzy priority queuing models, it has been shown that the proposed approach improves waiting time reduction, the management of the queue length and the efficiency of service provided. The model uses fuzzy logic to improve resource allocation, increase operational flexibility, and facilitate better decision-making in cases of uncertainty in priority-based service systems. The results showed that fuzzy priority queuing models

¹⁴ Sharma, P., Patel, R., & Verma, S. (2022). Modeling uncertain healthcare service systems using fuzzy priority queuing theory. *Healthcare Analytics*, 2, 100042. <https://doi.org/10.1016/j.health.2022.100042>

are more appropriate to the dynamic systems like healthcare systems, cloud computing, communication networks and other service-oriented applications.¹⁵

The proposed model offers many benefits, but there is potential for improvement through continued research by incorporating more sophisticated computational methods. There are several ideas for future research that should be explored, such as using Type 2 fuzzy numbers for higher levels of uncertainty, using machine learning techniques for dynamic parameter estimation and creating multi-server fuzzy priority queuing frameworks. Furthermore, real-time adaptive fuzzy scheduling mechanisms can be investigated to support intelligent decision making in a dynamically changing environment. The extension would further enhance the applicability and scalability of fuzzy queuing models for modern complex systems.

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¹⁵ Chen, X., Li, J., & Zhao, Y. (2023). Fuzzy queuing systems with priority discipline for cloud computing resource management. *Future Generation Computer Systems*, 143, 184–196. <https://doi.org/10.1016/j.future.2023.01.015>

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